<u>Chapter 7</u>: Techniques of Integration

<u>Section 7.4</u>: Integration by Partial Fractions

• Integration by Partial Fractions is the method you use to integrate a rational expression, like...

$$\int \frac{x^2 + 5x - 9}{2x^5 - x^4 + 7x^3 - 5x^2 + 8x + 1} \, dx$$





But we want to go backwards...



Idea...

$$\frac{x+5}{x^2+x-2} = \frac{2}{x-1} - \frac{1}{x+2}$$

So...

$$\int \frac{x+5}{x^2+x-2} \, dx = \int \frac{2}{x-1} - \frac{1}{x+2} \, dx$$

- Results:
- 1. The Fundamental Theorem of Algebra
- Any polynomial with real coefficients can be factored into
- degree 1 terms (linear terms) &
- irreducible degree 2 terms (irreducible quadratic terms)

- **Results:**
- 2. Partial Fractions Decomposition
- Any rational expression where deg(top) < deg(bottom) can be written as a sum of simpler rational expressions

Results: 2. Partial Fractions Decomposition

Notes:

• In the decomposition, if the denominator has a degree 1 term (the power outside of the parenthesis doesn't matter), then the top will be just a number A

Results: 2. Partial Fractions Decomposition

Notes:

• In the decomposition, if the denominator is an irreducible degree 2 term (the power outside of the parenthesis doesn't matter), then the top will be a degree 1 polynomial Ax + B

<u>Ex 1</u>: Write out the form of the partial fraction decomposition of the function below. Do not determine the numerical values of the coefficients...

$$x^3 - 2x + 7$$

 $(x-1)^4(x+5)(x+9)^3(x^2+1)^3(x^2+x+1)$

- Steps in an integration by partial fractions problem...
- 1. <u>Make sure you are integrating a rational expression</u>
- 2. If deg(top) \geq deg(bottom), use long division to divide the polynomials.
- 3. Factor the denominator on the remainder rational expression
- 4. Use the partial fractions decomposition on the remainder rational expression
- 5. Integrate each term

Integration by Partial Fractions <u>Ex 2</u>: Find ...

$$\int_{0}^{1} \frac{x^{2} + x + 1}{(x+1)^{2}(x+2)} dx$$

Integration by Partial Fractions <u>Ex 3</u>: Find ...

$$\int \frac{x^2 - x + 6}{x^3 + 3x} \, dx$$

Integration by Partial Fractions <u>Ex 4</u>: Find ...

$$\int \frac{x^3 - 3x^2 - 4x + 10}{x^2 + 2x - 3} \, dx$$

Integration by Partial Fractions <u>Notes on Factoring</u>: Factor by grouping

Factor $x^3 + x^2 + x + 1$

Notes on Factoring:

Rational roots theorem and synthetic division

Factor
$$x^4 - 2x^3 + 5x^2 + x - 22$$

Notes on Factoring:

Expressions of quadratic type

Factor
$$x^4 - 8x^2 + 15$$